INTERNAL RATES OF RETURN AND ENVIRONMENTAL REMEDIATION

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Abstract
Open pit mines, nuclear power plants, and the Trans-Alaska Pipeline are examples of projects with both a large initial cost and a large final environmental remediation cost. The result is a series of cash flows with two sign changes – the necessary condition for the present worth equation to have multiple roots.

Using examples, we analyze the conditions, consequences, and frequency of occurrence of multiple roots in this context. Finally, we will assess the significance of these results to theory and practice.

Introduction
It has long been recognized that some cash flow patterns have multiple roots for the present worth equation. The necessary condition is to have two or more sign changes in the cash flow series. The classic example is the "oil well problem." Adding an oil well to an existing field typically increases total recovery and shifts some oil recovery to earlier periods, so that the cash flow pattern is negative, positive, negative. Multiple roots for this problem are common.

Another class of problems with two sign changes is represented by open pit mines, nuclear power plants, and the Trans-Alaska Pipeline. These projects have both a large initial cost and a large final environmental remediation cost. The result is a cash flow with two or more sign changes – the necessary condition for the present worth equation to have multiple roots.

Past work on the multiple-root problem has focused on mathematical tests for the uniqueness of the root (Soper, 1959) and (Norstrøm, 1972), rather than on the circumstances where multiple roots occur. That work has established that multiple sign changes in the cash flow pattern are a necessary, but not a sufficient condition, for multiple roots. (Note that a necessary condition for a multiple root defines a sufficient condition for a unique root.) Other more restrictive conditions have been established (Bernhard, 1980), and some articles with algorithms have been published (Pasin and Leblanc, 1996).

More recent work has often focused on suggesting, supporting, or attacking measures, such as the modified IRR or overall rate of return, that "correct" for the multiple-root problem (Hajdasiński, 1997). Another approach suggests that the multiple root problem is not really a problem. Hazen (2003) claims "that contrary to current consensus, multiple or nonexistent internal rates are not contradictory, meaningless or invalid as rates of return. … when there multiple (or even complex-valued) internal rates, each has a meaningful interpretation as a rate of return on its own underlying investment stream."

Rather than focusing on the mathematical tests, we want to understand the conditions, consequences, and frequency of occurrence of multiple roots. In an earlier ASEM conference paper (Baker, Eschenbach, & Whittaker, 2003), we analyzed the most common situation that involves multiple sign changes. This comes from comparing equivalent annual measures (EACs and EAWs) of mutually exclusive alternatives with different length lives. This comparison assumes that the alternatives are repeated until a horizon that equals the least common multiple of the different length lives. We established that examples did exist with multiple roots, but for the examples we created the economic difference between the alternatives were generally small.

The intent of this paper is to examine a problem where we expected the multiple root issue to be of more concern. We want to begin placing reasonable bounds on the significance of the multiple-root problem in this context. Thus the results should be both theoretically and practically significant.

The Investment Hurdle
It is the large long term projects that have the greatest scrutiny. Mines and power plants generally have large start up costs and long and profitable lives. Thus they fit the classical profile of an investment project with an initial large negative cash flow followed by a lengthy series of positive ones. However, they differ from the classic investment project, because their final cash flow may be a salvage cost rather than a salvage value. Exhibit 1 illustrates the pattern of cash flows for such projects.

When analyzed over positive interest rates, the plot of the net present worth (NPW) has the same shape as that of a classic investment. Exhibit 2 illustrates the NPW curve that would normally be generated in analyzing investment projects (even those with a salvage cost). The NPW decreases as the interest rate increases.

Exhibit 1. Capital Project with Remediation Expense.

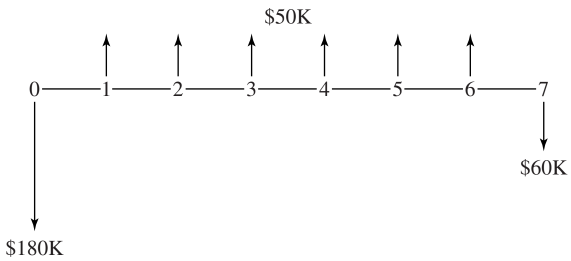
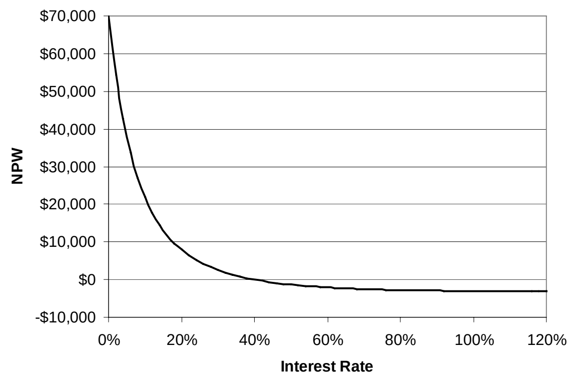


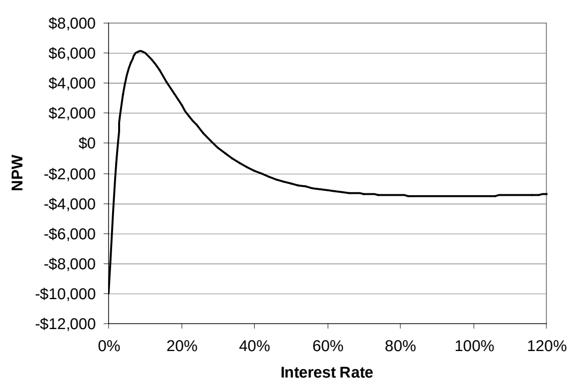
Exhibit 2. NPW vs. Interest Rate.



The rate of return is the rate at which the graph crosses the zero NPW axis. This graph reflects the familiar wisdom, that as the required rate of return increases, the project becomes increasingly less desirable (lower NPW) until the point where the NPW is negative and the project is not undertaken.

In some situations, the final environmental remediation cost may be large enough to create an obvious dual solution to the rate of return calculation as shown in Exhibit 3.

Exhibit 3. NPW Curve with Double Root.



If the accepted rule is that a positive NPW signifies acceptance and a negative one rejection; this produces the situation that for small rates the project should be rejected, for some middle range of rates it can be accepted, and for larger rates it should be rejected. If not an absurdity, this is at least a connundrum.

The intent of this paper is not to solve the riddle, but to explore its significance. To look what situations give rise to the dual rate curve, and to speculate on their likelihood.

Defining the Cash Flow Pattern

Coal mines, nuclear facilities, pipelines, power plants, and offshore platforms are examples of the problem being analyzed. They have a large initial cost, a long operating period with positive net revenues each year and a final cost to remove the facility, deal with waste products, and some level of environmental restoration.

For ease of analysis, the complexity of real examples has been simplified. Initial costs are assumed to occur at time 0 rather than being distributed over a several year start-up period. The annual net revenues are assumed to be constant over time, rather than a more typical declining gradient. Finally, the salvage cost is assumed to occur at the end of period N rather than being distributed over several years.

While the example is simplified from reality, we believe there is no loss of generality in the results. For spreadsheet analysis a first cost of $100,000 was chosen mainly so the number of digits would be reasonable for tables and graphs.

Then we selected two project lives reasonably representative of large mines or power plants, $N = 20$ years and $N = 50$ years. We then expressed the annual receipts, A, as a percentage of the initial cost, P; and the remediation cost, F, as a factor of the initial cost P. Thus, all cash flows would scale to match the scale of the assumed first cost.

Defining the Cases for PW Results

Having defined the example, let us now define the cases for the PW curve that result as we vary the example's parameters.

Case A. The base case, and familiar graph, with $N = 20$, $A = 20\%\ P$, and $F = 0$ is shown in Exhibit 4. In this instance, the IRR = 19.4% and the graph is positive in the 0% to 19.4% range. This graph has PW(0%) as a positive value and then the graph monotonically declines as the interest rate increases.

This is the classic example of the PW of an investment with a single sign change from negative initial cash flows to a series of positive cash flows.

However, in our case we must examine variations on this in more detail. For example, consider the case where the final salvage cost is ¼ of the initial first cost. As shown in Exhibit 5, this results in two roots for the NPW equation. They are −80.0% and 19.3%. In fact any final salvage cost that is negative leads to two sign changes in the cash flow pattern and according to Descartes Rule of Signs, there are two roots possible in the $(-1, \infty)$ interval. The maximum value of the NPW will occur between the two roots.

Exhibit 4. Case A Example NPW Curve.

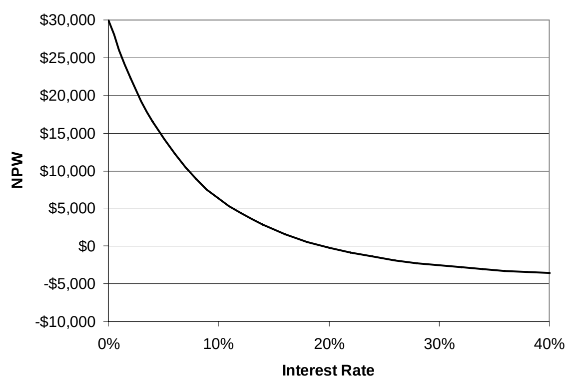


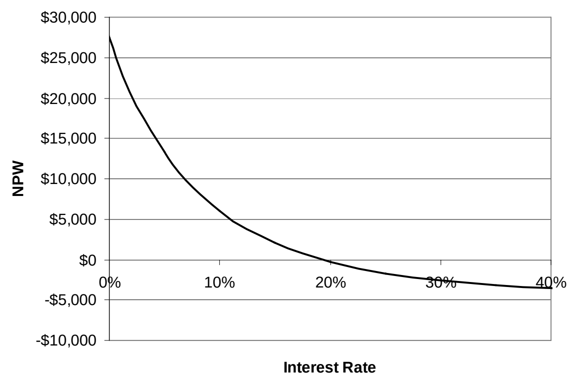
Exhibit 5. NPW vs. i with $F = -P/4$

Interest Rate	NPW
-85%	-2.95E+19
-80%	-6.26E+04
-75%	7.33E+14
-10%	$126,602
-5%	$57,481
0%	$27,500
5%	$13,316
10%	$6,050
15%	$2,057
20%	-$272
25%	-$1,697
30%	-$2,601
35%	-$3,190
40%	-$3,578
45%	-$3,834
50%	-$4,001

If however, as shown in Exhibit 6, this is graphed over the same region as in Exhibit 4, we find very similar results. Normal practice would be to ignore the very negative root and to simply use the 19.3% as an IRR.

Thus, we define Case A based on the presence of a single *positive* root, which would normally be defined as the IRR of the project. In our analysis, we divided this case into two subcategories. In Case A1 the graph of NPW will be monotonically decreasing from the value of NPW(0). In Case A2, the NPW curve will be maximized between $i = 0\%$ and the other positive root.

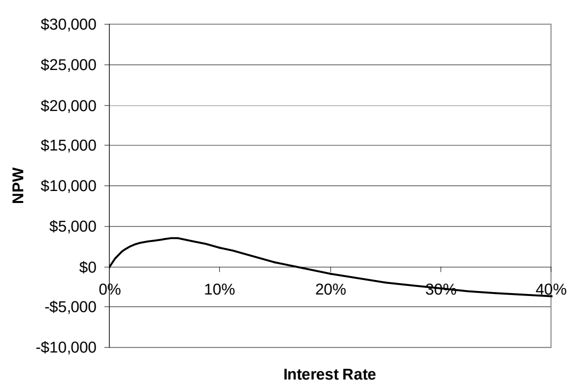
Exhibit 6. NPW vs. i for $i > 0$.



Case B. As the remediation cost is increased, the location of the negative root and the maximum value shift to the right. When the total cash flows equal 0, the first root is at 0%. Then the NPW increases with the interest rate to a peak and then decreases with a second positive root. At this point, the double root can no longer be ignored. Mathematicially speaking, we have a dual root situation with a root of 0 and one positive root. This pattern we will call case B.

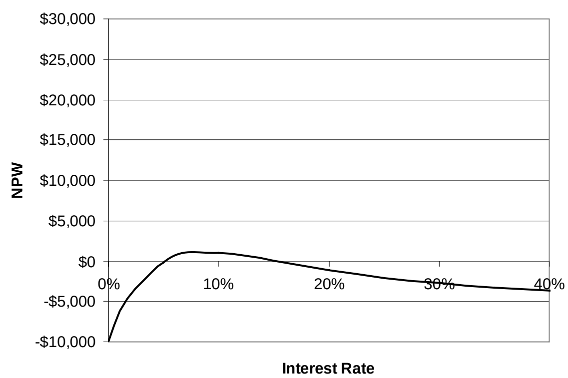
For this example, at a final cost equal to 3 times the intial cost the total cash flows equal 0. As shown in Exhibit 7, the two roots for our example are now 0% and 16.9%.

Exhibit 7. Case B Example NPW Curve.



Case C. Continuing to increase the size of the remediation cost shifts the graph to the right and produces the situation of an initial negative PW, increasing to a positive peak and then PW declining – that is two positive roots. As shown in Exhibit 8, the two roots for a remediation cost equal to 4 times *P* results are 5.2% and 15.3%. This pattern we call Case C.

Exhibit 8. Case C Example NPW Curve.

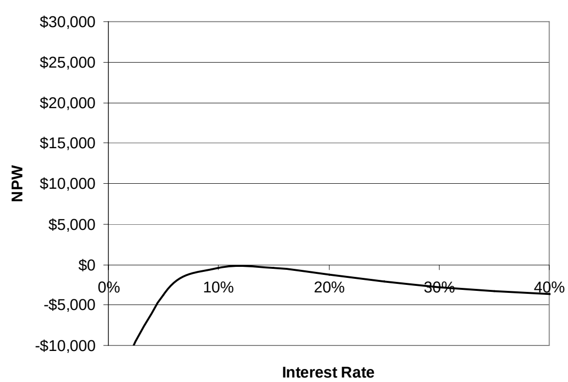


Case D. Continued increasing of the remediation cost pushes the peak down until the graph stays in the negative quadrants and there are no positive roots. This pattern is Case D.

The boundary between Cases C and D is defined in this example by using a multiplier of 4.44 times the first cost for the final salvage cost. This corresponds to a double root of 7.67%.

Exhibit 9 shows the situation with remediation cost equal to 5 times *P*.

Exhibit 9. Case D Example NPW Curve.



Analyzing Examples with *N* = 20 and *N* = 50

The tables and accompanying graphs in Exhibits 10 – 13 show the ranges of the different patterns. Notice that with *N* = 20, the annual revenue must be at least 5% of the first cost to repay the initial investment. However, if the annual revenue is less than 10%, we have to deal with cases where both roots are negative. As examples like these would be rejected out of hand, we've chosen to restrict our analysis to cases where the annual revenue is at least 10% of the first cost.

Exhibit 10. *F/P* Breakpoints for *N* = 20.

A%=	A1/A2	B	CD
10%	1	1	1
11%	1.2	1.2	1.2001
15%	1.6	2	2.31
20%	2.3	3	4.73
25%	2.8	4	9.38
30%	3.43	5	18.04
35%	4	6	33.86
40%	4.6	7	63.23

Exhibit 11. Case Regions for *N* = 20

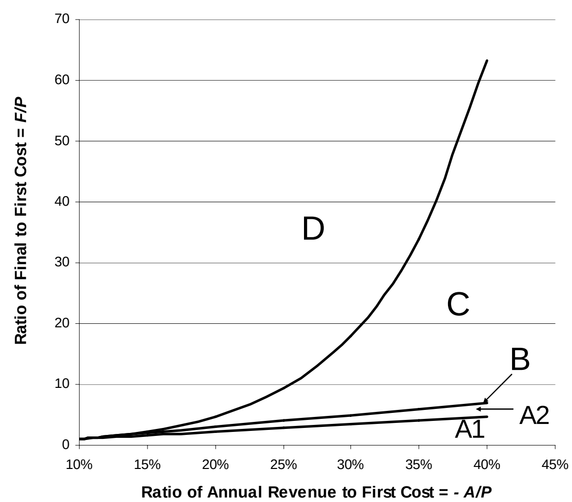
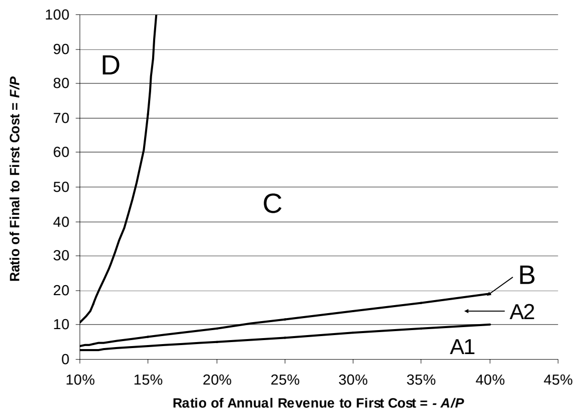


Exhibit 12. *F/P* Breakpoints for *N* = 50

A%=	A1/A2	B	CD
10%	2.55	4.0	10.61
11%	2.81	4.5	16.21
15%	3.83	6.5	71.00
20%	5.10	9.0	435
25%	6.38	11.5	2835
30%	7.66	14.0	16381
35%	8.93	16.5	100153
40%	10.21	19.0	528083

The range of values would make the graph relatively useless so an upper limit of 100 is used for the *F/P* ratio to graph these results.

Exhibit 13. Case Regions for $N = 50$.



Discussion and Reflection

The exhibits produce some interesting insights when combined with the observed trends in environmental legislation and corporate responsibility.

Looking first at the two ends, Case A and Case D. It can be seen that the range over which the classic investment curve, Case A, holds is really quite extensive. A 20 year project, for example a mine, with net revenues in the range of 20% of initial cost, remains a classic and viable project (i.e. an attractive IRR of 18%) even when faced with cleanup costs as large as twice the initial investment! For a 50 year project with the same returns, the cleanup costs can be a factor of six times the initial investment.

On the other hand, Case D, a situation where there is never a viable project (i.e. no positive PW) is unlikely to ever be considered at the analysis stage. Only by accident are people likely to get into situations where the remediation costs are four plus times the original cost. Thus we drop this case from consideration.

Cases B and C are the dual rate problem with which we have wrestled in our earlier paper, and indeed, the one the the field of engineering economy have been constantly wrestling with. In our earlier paper, we looked at classic textbook example problems and demonstrated that the conditions for multiple roots are found in only a narrow range of variables. Here we look at what might be considered characteristic cash flows of actual projects and find that to produce a dual situation generally requires remediation costs of such magnitude, relative to the original cost of the project, that commisioning of the project is highly questionable.

Even in the era that produced Enron, it seems that corporate responsibility is not likely to include commissioning projects which face remediation costs that are 10 or more times their initial costs.

These results show that the problem needs to be of some concern, but that hopefully it has not resulted in many examples of building a poor project.

It is also clear that this work is only the beginning of research which should be extended to more complex problems of many types.

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