AN APPROACH TO INVESTMENTS VALUATION IN THE COLOMBIAN ELECTRICITY MARKET USING REAL OPTIONS AND STOCHASTIC CONTROL THEORY

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Abstract

In this paper an approach to valuate an investment in the Colombian Electricity Market (CEM) using real options and stochastic control theory is presented. In the deregulated power generation markets new valuation, capital budgeting and decision making techniques are required to increase competitiveness of the generators. Although Colombia has a deregulated market structure, these new techniques are still to be applied. A fuel thermal power unit is valuated by using Vollert's work (2003), under the CEM conditions. Some results are shown and discussed.

Introduction

In the Colombian electricity sector, major structural changes have been taking place since the beginning of the 1990's, following the international liberalization trend initiated by the United Kingdom, Chile, New Zealand, the Nordic countries, and some states of the USA.

Private investors, primarily involved in the trading and expansion activities, are extremely impacted by the high variability of the electricity price. Colombia is a country where almost 70% of the electricity is produced by hydraulic centrals and, in consequence, the system is very susceptible to extreme climatic conditions (for example, "El Niño" phenomena). In addition, the economic and politic conditions are very unstable, increasing the uncertainty not only in the short but also in the long term.

As result, the restructuring trend and the market conditions have brought out a lot of challenging research topics. In electricity markets, the research focus has been on the short-term operation. Possible exercise of market power (Arbeláez and Garcia 2002), traders training (UNALMED and ISA 1998) and risk management using portfolio theory (Correa 2001) are among the main aspects that have been studied in Colombia.

However, just recently long-term issues, as capital investments, have gained attention. The investors want to know how to maximize their investment value in a market characterized by high sunk costs, increasing uncertainty and mainly irreversible investments. Flexibility has been identified as *conditio sine qua non* in this kind of markets (Vollert 2003) and the real options approach has captured the interest of researchers (not yet practitioners) as a way of incorporating this concept in the decision making process.

Problem Statement

In the CEM, previous works have attempted to introduce the flexibility concept and have implemented a real options approach to valuate expansion investments (Arango 2001, Osorio 2002, Montoya 2003, Mora and Agudelo 2003). However, those works have considered just the individual effect of an option over the project value. Brennan and Schwarzt (1985), Triantis and Hodder (1990), Trigeorgis (1993), among others, have stressed the relevance of the contingent nature of the embedded options.

Due to the Colombian market structure (mainly private investors in the expansion activity) and the uncertain and volatile investment conditions (due to both physical and socioeconomic aspects), the implementation of a framework considering the effect of a portfolio of options over the project life cycle is required to support the investors' decision making process.

Literature Review

An early work in the applicability of the real options framework to valuate investments in natural resources is the one by Brennan and Schwarzt (1985), which applies real options to valuate investments in a copper mine. Later, Paddock et al. (1988) and Pyndick (1993) studied the valuation of leases for offshore petroleum and the investment in a nuclear power unit, respectively.

The increment of the capacity was initially studied by Pyndick (1988). In this case, the value of the firm is considered as having two components: the value of the installed capacity and the value of the option to modify that capacity. An optimality condition is achieved when the present value of the expected cash flow from a marginal unit of capacity is equal to the total cost of adding that unit. Many variations of this general framework have been studied by Dixit (1993), Dixit and Pyndick (1994), Abel and Eberly (1994), Dixit (1995), among others.

Triantis and Hodder (1990) propose an impulse control model (from the stochastic control theory) to valuate projects involving different operation modes and exogenous uncertainty. Kobila (1993) and Benavides (1995) study the incremental investment under a stochastic demand in the electricity market; the former applies stochastic control theory to study the expansion in hydraulic generation system. Vollert (2003) formalized Triantis and Hodder' work by using not only impulse control models but also optimal stopping models.

The valuation of investments using a real options approach in the Colombian electricity sector has been studied by Arango (2001), Osorio (2002), Montoya (2003), Mora y Agudelo (2003). Mainly, these works apply Dixit and Pyndick work (1994) to define the optimal investment rule and the value of the investment opportunity.

However, the contingent structure of embedded options and the effect of exogenous competition (when the options are not exclusive to only one firm and the firm has to evaluate the value of waiting against getting a lesser value due to competitors actions) have not been studied for the CEM. This work makes use of Vollert's formalization of Triantis and Hodder's stochastic control framework to valuate an investment project in the CEM, considering multiple embedded options and operation under exogenous competition.

Model

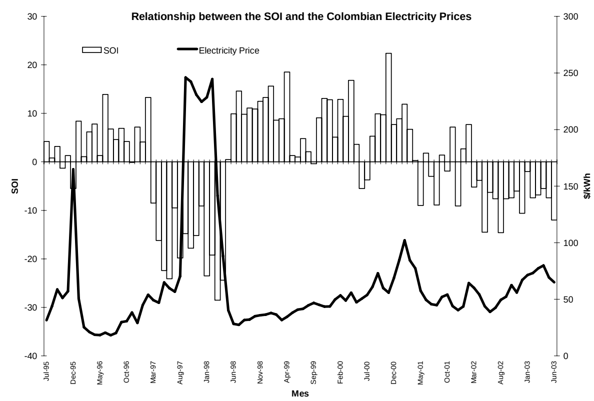
In this section of the paper, Vollert's model (2003), considering the conditions at the CEM, is implemented. An investment in a power fossil-fueled steam (thermal) unit is considered initially by using the traditional Net Present Value (NPV). Next, the flexibilities given by the proposed approach are analyzed. Timing and intensity are decisions to be considered in this kind of investment as well as the random nature of the electricity prices at the CEM. These elements are modeled in the next sections.

Electricity Prices. As mentioned before, the electricity price is the main source of uncertainty at the CEM. In this work this price is modeled as a General Brownian Movement (GBM) switching between two states, to replicate some physical conditions of the market. In this sense, two different periods are considered, one of high prices and the other one of low prices. Initially, a period of low prices is considered. After an interval of time t, the price jumps to a period of high prices.

Two considerations must be taken into account. First, the nature of the variable describing the change between periods, and second, the nature of the stochastic processes describing the electricity prices in both periods.

In order to model these considerations, a relationship between the "El Niño" Southern Oscillation Index (SOI) and the electricity price must be established. Exhibit 1 shows such a relationship. There is a correlation index of -0.65 between these two variables. Therefore, the SOI can be used to model some features of the electricity price such as the duration of the periods of both high and low prices and, the behavior of it in the two periods.

Exhibit 1. Relationship between the SOI and the Electricity Price.



By using the SOI's monthly data and assuming that a SOI's value of less than –10 can be interpreted as the price being in a period of high values, the duration of the high prices period as well as the probability of changing from one period to another can be obtained. The equation 1 shows the probability of changing from a period of low prices to one of high prices. Once in the period of high values, the price is kept there for a period of 10 months, and then it goes back to the low values period.

$$\begin{aligned} P(\tau \le t + dt/\tau > t) &= \frac{P(\tau \in (t, t+dt])}{P(\tau > t)} \\ &= \frac{P(\tau \in (t, t+dt])}{1 - P(\tau \le t)} \\ &= \frac{\varphi\left(\frac{t-\mu_\tau}{\sigma_\tau}\right)}{1 - \Phi\left(\frac{t-\mu_\tau}{\sigma_\tau}\right)} dt = l(t)dt \end{aligned} \qquad (1)$$

where $\varphi(x)$ y $\Phi(x)$ are the density function and the normal probability function respectively, and τ, μ_τ y σ_τ are the duration of the period of low prices, its mean (42 months) and its standard deviation (10 months).

On the other hand, the process for the electricity price is assumed as a General Brownian Movement (GBM):

$$dpe_t = \mu pe_t dt + \sigma pe_t dB_t, \quad pe_t \geq 0 \tag{2}$$

where pe_t is the electricity price at time t, μ is the drift parameter, σ is the constant volatility, and B_t is a Weiner process. Finally, the processes for the high and low prices periods can be defined respectively:

$$\begin{aligned} dpe_t &= \mu_H pe_t dt + \sigma pe_t dB_t \\ dpe_t &= \mu_L pe_t dt + \sigma pe_t dB_t, \quad pe_t \geq 0 \end{aligned} \tag{3}$$

where μ_H and μ_L are the drift parameters for the high and low prices periods respectively.

Costs Function. As mentioned before, a power thermal unit is considered in this paper. Such unit converts a particular fuel into electricity at a rate H. The expected payoff P per each MWh of electricity is (Tseng and Barz 2002):

$$P = pe - H * pf \tag{4}$$

where pf is the fuel price. In this paper pf is assumed as a constant. In this sense, $H(q)$ represents the quantity of heat required to generate q MW of power and, it is assumed constant at all levels of generation. Tseng and Barz (2002) defined H as a quadratic function of q and, based on the fuel price, the cost can be defined as follows:

$$\begin{aligned} C(q_t, pf) &= H(q_t) * pf \\ &= (a_0 + a_1 q_t + a_2 q_t^2) * pf \end{aligned} \tag{5}$$

Also, in order to construct the power unit an initial investment outlay must be paid. This investment is based on the installed capacity M of the unit as follows:

$$II(t, pe_t, M) = a_3 M \tag{6}$$

where $a_3 > 0$ denotes the constant cost of one unit of capacity. Once the power unit is constructed, it can be sold for its scrap value determined by both the contraction and expansion costs. These costs are defined as follows:

$$EI(t, pe_t, M_0, M_1) = a_4 (M_1 - M_0) + a_6 \tag{7}$$

in the case of capacity expansion ($M_1 > M_0$) and

$$CI(t, pe_t, M_0, M_1) = a_5 (M_0 - M_1) + a_6 \tag{8}$$

in the case of capacity reduction ($M_1 < M_0$). Again, a_4 and a_5 are assumed to be positive and constant. a_4 denotes the unit cost of capacity expansion (usually higher than the initial cost a_3). Depending on the situation, the constant a_4 may take a positive value (unit cost of desinvestment) or negative (scrap value of one unit of capacity), meaning an income on the payoff equation. Finally, a_6 denotes the fixed cost of making a change in capacity.

In this paper it is assumed that every capacity decision takes effect immediately. However, the effect of time lags on project values, optimal investment, and operating policies can be easily modeled (Vollert 2003).

Static project value with fixed capacity. The initial investment has to be done at time 0. The only sources of flexibility for the power unit are to adjust the production capacity and to choose the initial capacity. Also, the two different periods of electricity prices have to be considered into the analysis. In this case, the ability of the unit to adjust its production rates immediately, at no extra cost, is equivalent to maximizing the instantaneous cash flow function f with respect to $q \in [0, M]$ and q can be defined based on the electricity and fuel prices as follows:

$$q^* = \begin{cases} 0 & pe < a_1 pf \\ \dfrac{pe - a_1 pf}{2 a_2 pf} & a_1 pf \leq pe \leq pf(2a_2 M + a_1) \\ M & pe > pf(2a_2 M + a_1) \end{cases} \tag{9}$$

The corresponding optimal cash flow function f for each one of the three cases can be defined as follows:

$$f = \begin{cases} -a_0 pf & pe < a_1 pf \\ \left[\left(\dfrac{pe - a_1 pf}{2 a_2 pf}\right)\left(\dfrac{a_1(pf - 1)}{2 pf}\right)\right] - a_0 pf & a_1 pf \leq pe \leq pf(2a_2 M + a_1) \\ M & pe > pf(2a_2 M + a_1) \end{cases}$$

(10)

Note that the cash flow is negative if the unit does not generate power.

Next, the project value of the investment opportunity without options is analyzed. Let VP_H be the project value in the high prices period and VP_L the project value in the low prices period. The state of the system can be defined by using a vector X_t with three components; time t, electricity price pe (stochastic variable), and the initial capacity M.

$$X_t = \begin{pmatrix} t \\ pe_t \\ M \end{pmatrix} \tag{11}$$

The action space Z consists of the choice of the initial capacity. In general, there are some constraints for the possible range of capacity values. In particular, in the CEM the values are constrained by market regulations. In order to participate in the competitive market, the power unit's capacity must be greater or equal than 20 MW (Energy and Gas Regulation Commission 1995). Due to competitive rules, aimed to avoid power market, and technological constraints, the power unit's capacity must be less or equal than 1000 MW. Therefore Z = [20, 1000], and the initial value of $X_0 = (0, pe_0, M)$.

On the other hand, VP_H and VP_L can be expressed as partial differential equations (PDE) as follows:

$$\frac{1}{2}\sigma^2 pe^2 \frac{\partial^2 VP_L}{\partial pe^2} + \mu_L pe \frac{\partial VP_L}{\partial pe} + \frac{\partial VP_L}{\partial t} - (r + l(t))VP_L + f + l(t)VP_H = 0 \tag{12}$$

$$\frac{1}{2}\sigma^2 pe^2 \frac{\partial^2 VP_H}{\partial pe^2} + \mu_H pe \frac{\partial VP_H}{\partial pe} + \frac{\partial VP_H}{\partial t} - rVP_H + f = 0 \tag{13}$$

where r is the annual yield of the electricity in the MEC.

At terminal time T both in the high and low prices periods, the initially installed capacity M is sold for its scrap value. Hence, the two terminal conditions are:

$$\begin{aligned} VP_H &= -a_4 M - a_6 \\ VP_L &= -a_4 M - a_6 \end{aligned} \tag{14}$$

Finally, since the initial capacity can be chosen, M_0 has to be found such that, the expected profit of the power unit minus the initial investment outlay necessary to build the unit with capacity M_0, is maximized as follows:

$$\underset{M \in [M_{Min}, M_{Max}]}{\text{Max}} \left\{ VP_L(0, pe_0, M) - a_3 M \right\} > 0 \tag{15}$$

It is assumed that the unit begins to operate in a period of low prices. If the objective function is negative, the project is rejected as in the classical NPV. In this case there is no real option since the optimal initial capacity is *a priori* fixed (Vollert 2003).

Timing and Intensity. If there is the strategic option to choose the timing and intensity of investment, the optimal strategy cannot be judged by using an NPV rule. Instead, a real option analysis has to be carried out to determine the expanded project value of the investment opportunity.

Now, the state of waiting can be defined. In this state there is no cash flow. Once the electricity price reaches a certain trigger value, pe_G, at time G the power unit will start to operate at an optimal capacity level, M_G, with a cost of $a_3 * M_G$. It appears to be better to invest in the high prices period but both cases have to be considered. Therefore, as for the project value, two options of waiting must be considered, one in the high prices period and the other one in the low prices period.

The value of the timing and intensity option, at time t, in the high prices period can be defined as follows:

$$VPW_H(X_t) = \underset{(G,M) \in [0,T] x [M_{Min}, M_{Max}]}{\text{Sup}} E\left(e^{-r(G-t)} VPW_H(X_G) - e^{-r(G-t)} a_3 M \right) \tag{16}$$

Similarly, the value of the timing and intensity option in the low prices period can be defined.

At a glance, this procedure checks at each time t the value of the electricity price pe_t in order to make a decision about starting operations. If the option value of further waiting is greater than the project value net cost of installing capacity, it is better to continue waiting. On the other hand, if the electricity price rises enough at time t and reaches a trigger level pe_G, the corresponding optimal capacity maximizing the project value minus investment costs is chosen.

Flexible Capacity. The next case to consider is the option to adjust capacity while the power unit is operating. The initial optimal capacity M_0 that maximizes the value of the unit can be chosen at time t_0. Additionally, capacity can be adjusted in a range between M_{Min} and M_{Max} by paying the switching costs EI and CI, whenever the electricity price is large enough.

This flexibility to adjust is desirable in the presence of large electricity prices fluctuations; allowing to cut the fixed costs incurred by maintaining large capacity at low demand. On the other hand, when electricity prices increase, the ability to expand capacity can be a source of additional profits.

Since capacity adjustments are reversible, the described situation can be modeled as a generalized switching option. In this case, M = 0 means to abandon the project.

The value of the project with flexibility in the high prices period can be defined as follows:

$$VPF_H(X_t) = \text{Sup}\ E\left(\int_t^T e^{-r(s-t)} f(X_s) ds - \sum_{i:t\le\theta_i\le T} e^{-r(\theta_i - t)} EI/CI\right) \quad (17)$$

where θ_i is a finite sequence of stopping times and corresponding impulse controls.

Similarly, the value of the project with flexibility in the low prices period can be defined.

This equation means that whenever changing the current level of capacity is optimal, the new capacity is chosen such that it maximizes the new net project value over all possible upward or downward adjustments (Vollert 2003).

Also, in this option, there exists an optimal expansion or contraction policy for each time t and at each capacity level M_t which is given not only by the current capacity but also by the electricity price.

Timing, Intensity and Flexible Capacity. Since there is not commitment to invest immediately, the timing and intensity option must be considered in combination with the flexibility capacity option.

From a mathematical point of view, the only difference from the timing and intensity option is that now the value of the project with flexible capacity minus investment cost is taken into account instead of the value of the project with fixed capacity minus investment cost (Vollert 2003).

The value of the project with timing, intensity and flexible capacity in the high prices period can be defined as follows:

$$VPWF_H(X_t) = \underset{(G,M)\in[0,T]x[M_{Min},M_{Max}]}{\text{Sup}} E\left(e^{-r(G-t)} VPF_H(X_G) - e^{-r(G-t)} a_3 M\right) \quad (18)$$

In the next section, based on the implementation of the model, some results are provided and some analysis are made.

Results

By using finite difference methods, implemented on Visual Basic, the model has been applied to a power fossil-fueled steam (thermal) unit at the CEM.

The method creates a rectangular discretization of the state space (time, electricity price and capacity). A number of equally spaced times between zero and the end of the time horizon T are chosen. Also, a number of equally spaced electricity price levels and capacity levels are chosen as shown by equation 19.

$$\begin{aligned} \Delta_t &= \frac{T}{N_t} \\ \Delta_{pe} &= \frac{(pe_{Max} - pe_{Min})}{N_{pe}} \\ \Delta_M &= \frac{(M_{Max} - M_{Min})}{N_M} \end{aligned} \quad (19)$$

The resulting grid consists of $(N_t+1)(N_{pe}+1)(N_M+1)$ points. Exhibit 2 shows these values.

Exhibit 2. Discretization of the state space.

Variable	Unit	Name	Min	Max	N	Δ
Electricity Price modified	$/MWh	pem	10.3	13.1	28	0.4
Capacity	MW	M	0	1000	200	5
Time	year	t	0	20	200	0.1

The minimum capacity was defined at zero in order to take into account the option to abandon the project.

As shown by Trigeorgis (1996), the explicit finite difference scheme is better due to its computational efficiency and its ability to overcome stability and convergence problems. Additionally, Brennan and Schwartz (1978) proposed to use this scheme with the natural logarithm of the stochastic underlying variable in order to avoid the violation of the no negativity and unitary value of the probabilities involved in the solution process.

In order to solve the equations 12 or 13, the value of the project VP can be solved as follows (Trigeorgis 1996):

$$V_{i,j,k} = \frac{p_j^+ V_{i+1,j+1,k} + p_j^0 V_{i+1,j,k} + p_j^- V_{i+1,j-1,k} + f_{i,j,k}\Delta_t}{1 - r\Delta_t} \quad (20)$$

where

$$\begin{aligned} p_j^+ &= \frac{1}{2}\sigma^2 j^2 \Delta_t + \frac{1}{2}\mu_{H/L} j \Delta_t \\ p_j^0 &= 1 - \sigma^2 j^2 \Delta_t \\ p_j^- &= \frac{1}{2}\sigma^2 j^2 \Delta_t - \frac{1}{2}\mu_{H/L} j \Delta_t \end{aligned} \quad (21)$$

By considering the log-transformed problem equation 21 becomes:

$$p_j^+ = \frac{\Delta_t}{2\Delta pem^2}\sigma^2 + \frac{\Delta_t}{2\Delta pem}(\mu_{H/L} - \frac{\sigma^2}{2})$$
$$p_j^0 = 1 - \frac{\Delta_t}{\Delta pem^2}\sigma^2 \qquad (22)$$
$$p_j^- = \frac{\Delta_t}{2\Delta pem^2}\sigma^2 - \frac{\Delta_t}{2\Delta pem}(\mu_{H/L} - \frac{\sigma^2}{2})$$

where *pem*, the electricity price modified, is *ln(pe)* and its Δpem is as defined at Exhibit 2. Hull (1997) pointed out that the explicit finite difference method attains its highest numerical efficiency for $\Delta pem = \sigma(3\Delta_t)^{1/2}$.

Exhibit 3 shows the value of the parameters used to run the implementation of the model. Exhibit 4 shows the project and option values obtained by using this set of parameters.

Exhibit 3. Parameters used.

Parameter	***Value***	***Unit***
Time horizon	20	Years
Initial electricity price (June 2003)	65,030	$/MWh
Logarithm of the initial electricity price (June 2003)	11.1	$/MWh
Electricity yield	1.3%	Annual
Riskless interest rate	7.0%	Annual
Volatility	76.9%	Annual
Average duration of the period of low prices	3.5	Years
Standard deviation period of low prices	1.33	Year
Fuel price	2.9	US $/MMBTU
Interchange rate June 2003	2,817	$/dollars
Fuel price	8,170	$/MMBTU
Heat Rate (H)	7	MMBTU/MWh
Days per year	365	
Hours per day	24	
Efficiency	100%	
Generation factor	8,760	MWh/year
Investment cost	650	US $/MW
Investment cost	2	Millions of pesos/MW

Note that the project with flexibility capacity option has the same value than the project without timing and intensity option.

Recommendations and Further Research

This work is a rough attempt to apply Vollert's work (2003) for the valuation of investment opportunities in the CEM. Many aspects need to be improved, from both the methodological aspect and the theoretical side. Some of those aspects are depicted as follows:

The cost structure must be revisited. The parameters of the Tseng and Barz's function have to be estimated more accurately. The results presented in this work might be underestimating the cost structure of a thermal power unit in Colombia. This fact might lead to the underestimation of the embedded options as well as the project value; which might give wrong signals to investors about when and how to intervene their system. Also, the efficiency of the unit has to be calculated based on a set of specifications that are roughly depicted in this analysis. Investors have to understand not only the physical characteristics of the unit but also the nature of the fuel prices and look for potential colinearities and correlations between those and the electricity price.

Exhibit 4. Project and option values.

	Immediate Investment	**Timing and Intensity Option**
Static Project	**Value** = 526,725 millions **Capacity** = 1000 MW	**Value** = 1,274,420 millions **Capacity** = 1000 MW **Trigger price** = 147,267 $/MWh **Starting time** = 3.3 years
Flexible Capacity	**Value** = 1,274,781 millions **Initial capacity** = 5 MW **Final capacity** = 1000 MW **Trigger price** = 147,267 $/MWh	**Value** = 1,274,781 millions **Initial capacity** = 5 MW **Final capacity** = 1000 MW **Trigger price** = 147,267 $/MWh **Starting time** = 0 years

Another issue to review is the nature of the volatility. In the analyzed model, it was assumed the volatility was a constant over time. Based on historical data (approximately eight years of monthly data), it is possible to conclude that further analysis must include a stochastic model for the behavior of the volatility, specially before, during and post extreme events. Also, based on historical data and some preliminary studies, the assumption of a Geometric Brownian Motion for the electricity price might be no accurate. Although until now, the models proposed to represent the nature of the electricity price in the CEM have performed poorly, it is required to understand the underlying process and to integrate it to the control framework proposed by Vollert (2003).

Even though the CEM has undergone through a deep restructuring, the market still operates under incomplete information. Then, the cost of the information must be introduced within the valuation framework.

Strategic aspects as the effect of delaying a decision or participating in the regulated market were omitted by simplicity. However, in order to develop a more robust model to support decision making processes, it will require the incorporation of those and other strategic aspects.

Finally, the real options approach and moreover, its formalization through the use of control models is far from being used as a methodology to support the agents decision making process. More work has to be done in the validation of the applicability of this kind of analysis. Then, two big challenges emerge here for the engineering managers. First, the practical and strategic implications of the use of control theory and the real options approach must be understood as well as the theoretical aspect behind those. In other words, engineering managers must be able of tailoring the approach to the specific needs of the industry or company without oversimplifying the assumptions behind the implementation.

Also, this approach must be incorporated within an existing decision support system. The approach itself is not a decision support system for long term investments. It is intended to value the managerial flexibility; however, at certain points that flexibility is worthless (Huchzermeier and Loch, 2001). The success on using this approach relies on the information flow feeding the models and how those results are related to a bigger context. That is the biggest challenge for the practitioners.

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