MULTIPLE OBJECTIVE DECISION MAKING USING ESTIMATED UTILITY FUNCTIONS

Sampson E. Gholston, Ph.D., The University of Alabama in Huntsville
Alisha D. Youngblood, Ph.D., The University of Alabama in Huntsville

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Abstract

Decision making in an organization is more complex when multiple criteria are used in decision making models. Multiple criteria decision making involves the selection of the "best" solution for a system involving multiple conflicting objectives. In order to use some types of multiple criteria decision models an appropriate utility function for each of the criteria must be determined based on data elicited from the decision maker. The elicitation of information from the decision maker to determine an appropriate utility function for each criterion can be a lengthy, difficult, and subjective process. This paper highlights the development of a decision model that incorporates the approximation of utility functions using a minimum amount of data from the decision maker and ranks the criteria so that the difficulties inherent in weighting can be avoided.

Introduction

Optimization in mathematical programming generally means maximizing (or minimizing) an objective subject to a set of constraints. Unfortunately, most decision making problems involve the simultaneous consideration of several different conflicting objectives. By nature of the conflicting objectives, finding an "optimal" solution to these design problems presents some challenges for the person or persons responsible for making the decision. Due to the conflicting nature of the objectives, an optimal solution that simultaneously maximizes all objectives is usually not attainable. There are several solutions which are called compromise solutions which are candidates for the final solution. They are called compromise solutions because the only way to improve the value of any objective would be to compromise the value of at least one of the other objectives. Multiple objective modeling reflects the decision process where several conflicting objectives have to be satisfied. The task then becomes choosing the best compromise solution.

The person or persons responsible for choosing the best compromise solution are referred to as decision makers. One of the most common complications for decision makers faced with real-world applications is the presence of multiple, conflicting objectives (Graves et al., 1992). When there are two or more objectives, then generally there is no agreement on how to proceed in the solution process. The ultimate goal in the approaches to solve multiple objective decision making problems is to help a decision maker find his/her most preferred solution for the decision making problem. Decision makers often rely on Multiple Criteria Decision Making (MCDM) tools to assist them in choosing the best alternative solution. Since there are usually a large number of solutions that are candidates for the best compromise solution, MCDM tools must assist the decision maker in maximizing his/her preferences. The solution that maximizes the decision maker's preferences is known as the best compromise solution.

The techniques and algorithms used in multiple criteria decision making are only aids to assist the decision maker in the decision making process. Making the decision is a managerial task which should never be automated (Stewart, 1992). Decision makers are able to evaluate the consequences of their decisions using MCDM tools (Olson, 1992).

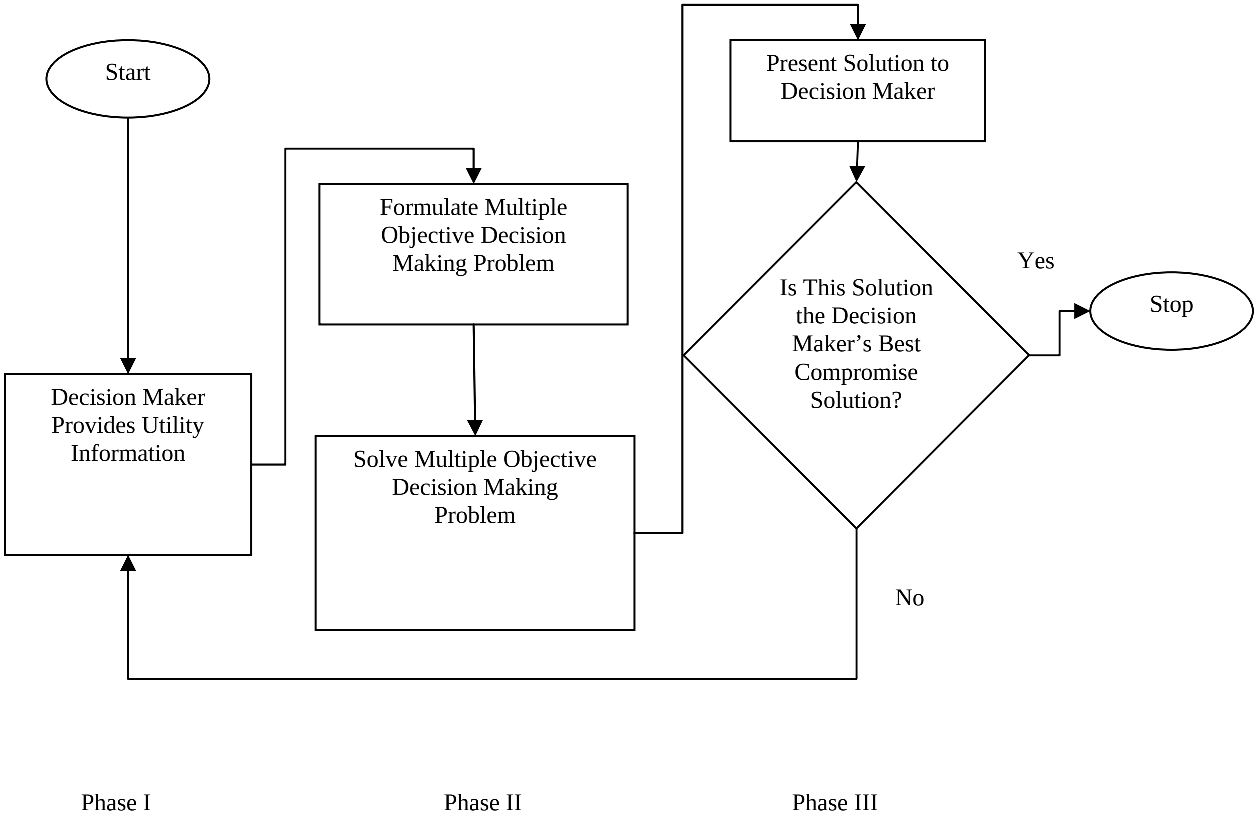
A Multiple Objective Optimization Procedure

Gholston (1999) developed a multiple objective optimization procedure. This procedure required the decision maker to rank each objective in order of importance and find a solution based on this ranking. The procedure below is a modification of that original estimating procedure. The modification involves a utility function estimation procedure developed by Youngblood (2003). There are three phases in the developed procedure, which are:

Phase I - Determine Decision Maker's Utility
Phase II - Formulate and Solve MCDM Problem
Phase III - Phase IV – Determine Compromise.

The three phases lead decision makers to their best compromise solution. The procedure developed by Gholston (1999) was perhaps biased by the initial starting solution. By using utility functions, the final solution is not biased by the initial solution. For that reason the procedure was modified to utilize utility functions. Exhibit 1 shows a flowchart of the modified procedure.

Exhibit 1. Flowchart of multiple objective optimization procedure.



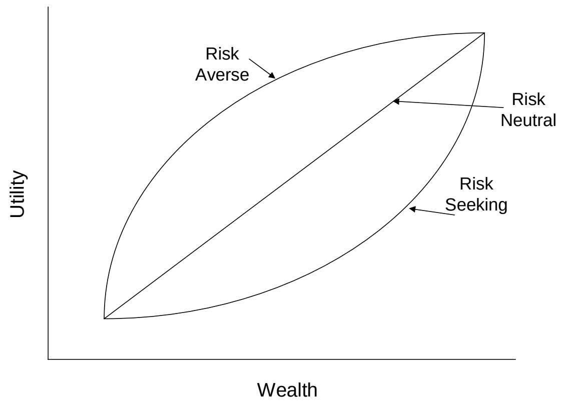
In **Phase I** the decision maker's utility is determined using a procedure developed by Youngblood (2003). Although it is common practice to determine expected values when choosing between scenarios with anticipated outcomes and associated probabilities, frequently this results in decisions that may not seem "intuitively appealing" (Clemen, 2001). An individual's risk attitude can often be assessed by having the person choose between several pairs of gambles and sure amounts that may be greater than or less than the expected value of the associated gamble. In general, if a person prefers a specified amount over a gamble with a greater expected value, then the individual would be considered risk averse. On the contrary, if an individual prefers a gamble with a lower expected value than a certain and specified amount, the individual would be considered risk seeking. Individuals who express indifference between a sure thing and a gamble of equal expected value would be considered risk neutral.

A decision maker's individual risk profile is a subjective assessment of the person's attitude towards risk. This can be represented mathematically through the use of utility functions. Although these functions are unique to a decision maker's attitude towards a particular attribute, there are certain characteristics that remain consistent for all utility functions. One difficulty with the subjective derivation of utility functions often noted in research is the overemphasis on events with a low probability (Quiggin, 1993).

When developing a utility function, a range of values depicting both worst and best case scenarios for each attribute is determined. In most situations, one end of a range is considered "best" and the other "worst." However, occasionally the "best case scenario" is a value somewhere in the middle with each end considered equally bad (Keeney, 1975.) Multi-attribute utility theory allows for cases such as these. By definition of utility functions, the worst-case value for a metric is considered to have a utility value of zero, U(worst-case value) = 0, and the best-case value has a utility equal to one, U(best-case value) = 1.

It is then necessary to determine a function joining the two points, depending on whether the decision maker has a risk-seeking, risk-averse, or risk-neutral attitude regarding the trade-offs for that objective. These utility functions are a continuous zero-to-one function, but the shape is determined by the decision maker's risk profile with respect to the individual attributes. If the decision maker's attitude is considered to be risk neutral, then the utility function will produce a straight line. In the case of a risk averse attitude, the curve will be convex, while risk seeking will produce a concave curve (Taha, 1997). Examples of positively-sloped utility curves associated with these risk profiles can be seen in Exhibit 2.

Exhibit 2. Risk Attitudes.



Each *risk-neutral* utility function used in a utility model is the straight line segment joining the two end points (best-case and worst-case) in a range of likely values for a metric. The necessary parameters are the two end points and an indication as to whether the slope is positive or negative. Positively-sloped utility functions model measurements where improvement occurs when the measurement increases like perfect order rates. Negatively-sloped utility functions model measurements where improvement occurs when the measurement decreases like cost per operation.

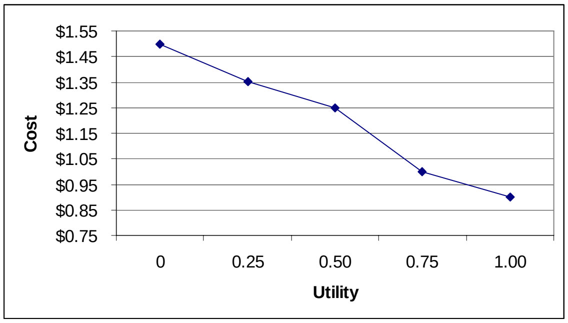
Exhibit 3. Utility Functions.

Performance Level	Cost	Utility Value
Poor	\$1.50	0.00
Below Average	\$1.35	0.25
Average	\$1.25	0.50
Above Average	\$1.00	0.75
Exceptional	\$0.90	1.00

Risk-seeking and *risk-averse* utility functions require more information than *risk-neutral* functions. To generate these shapes, spline fitting is used. The initial two data points are the same best-case and worst-case values determined for the corresponding *risk-neutral* function. The remaining three points for the spline fitting are generated as seen in Exhibit 3. The decision maker does not need to understand his risk profile to generate this function.

When the sample data elicited in Exhibit 3 is plotted, it results in the graph shown in Exhibit 4. As can be seen in this graph, utility functions do not necessarily have to be consistently concave or convex, but can show different risk attitudes at different points throughout the range of values for the metric.

Exhibit 4. Utility Functions Graph.



In **Phase II** the multiple objective problem is formulated from the information provided by the decision maker. The decision maker provides the utility information by providing five values for each objective. The multiple objective problem is converted to a single objective problem using the following methodology.

For objectives to be maximized, a utility value is calculated using the following equation

$$U_i = \left(\frac{Y_i - Y_{MIN}}{Y_{MAX} - Y_{MIN}} \right), \; Y_{MIN} \leq Y_i \leq Y_{MAX} \quad (1)$$

where,

U_i is the utility value between 0 and 1

Y_i equals the value of objective function at the current solution

Y_{MIN} and Y_{MAX} equals the minimum and maximum values of each objective, respectfully, subject to the constraints.

For objectives to be minimized a utility value is calculated using the following equation

$$U_i = \left(\frac{Y_{MAX} - Y_i}{Y_{MAX} - Y_{MIN}} \right),\ Y_{MIN} \le Y_i \le Y_{MAX} \quad (2)$$

These utility values are scaled between 0 and 1, and a final score is assigned based on the utility function that is given by the decision maker. These values map back to utility values, which directly relate to the information supplied by the decision maker.

The problem formulation is

$$\text{Maximize } \sum U_i$$

subject to the constraints. The single objective problem could then be solved using single objective optimization procedures, such as linear programming.

Providing the values for each objective is not as difficult as providing explicit tradeoff information for each objective. Shin and Ravindran (1991) indicate that pairwise (ordinal) comparisons are the easiest type of interactive style for the decision maker. This procedure allows decision makers to consider each objective individually initially. If the solution is unacceptable, the decision maker will have to supply new values for the objectives with the understanding that improvements in unsatisfactory objectives can only be achieved by compromising other objectives.

In **Phase III** the decision maker evaluates the solution. By definition this nondominated solution is a candidate for the best compromise solution. The current solution is presented to the decision maker. The decision maker determines if the solution is acceptable. If it is, the procedure stops. Otherwise, the decision maker provides new values for the utility functions.

Multiple Objective Example

In order to demonstrate the procedure developed in this research, a production planning example was used. The example was conducted with a modification of an example used by Fogiel (1983). There were two objectives to the production planning decision making example used here. The objectives were to minimize production cost, and maximize quality levels. The decision variables for this multiple objective problem include

x_1 = number of units produced on first shift,
x_2 = number of units produced on second shift,
x_3 = number of units produced by subcontractor, and
x_4 = number of units produced by temporary employees.

The two objective problem formulation is

Minimize cost: $30x_1 + 20x_2 + 20x_3 + 24x_4$
Maximize quality:
$(0.98x_1+0.95x_2+0.85x_3+0.80x_4)/(x_1+x_2+ x_3+ x_4)$,

where it cost $30 dollars per unit for product produced on the first shift and 98% of the product produced on the first shift is of good quality. The equations above illustrate the relationships between quality, cost and number items produced.

The constraints for this problem are given below.

Production hours: $2x_1 + 2x_2 \le 100$,
Temporary contract: $x_4 \ge 10$, and
x_1, x_2, x_3, and $x_4 \ge 0$.

The decision maker supplies the values used to determine the utility function. For demonstration purposes suppose the decision maker supplies the utility values shown in Exhibit 5. The problem is then converted to a single objective optimization problem by maximizing the sum of the utility values for each objective. This guarantees that an efficient solution is found and presented to the decision maker. The decision maker then determines if this solution is the best compromise solution.

Exhibit 5. Utility Functions for First Iteration.

Performance Level	Quality Level	Total Cost	Utility Value
Poor	5.0%	$1,200.00	0
Below Average	10.0%	$1,100.00	0.25
Average	50.0%	$900.00	0.50
Above Average	86.0%	$450.00	0.75
Exceptional	88.0%	$240.00	1.00

The single objective problem formulation is giving below.

Maximize: $U_{quality} + U_{Cost}$

Subject to:
Production hours: $2x_1 + 2x_2 \le 100$,
Temporary contract: $x_4 \ge 10$, and
x_1, x_2, x_3, and $x_4 \ge 0$.

For example, maximize the utility score for quality plus the utility score for cost is shown in the problem formulation above and the utility function supplied by the decision maker is in Exhibit 5. The solution is found using linear programming and is given below.

x_1 = 4 units,
x_2 = 0 units,
x_3 = 0 units, and
x_4 = 10 units.

The values for the objective functions are:

$$\text{Cost} = (30x_1 + 24x_4) = (30 \times 4) + (24 \times 10)$$
$$= \$360.00$$

$$\text{Quality} = (0.98x_1 + 0.80x_4)/(x_1 + x_4)$$
$$= (0.98\times4 + 0.80\times10)/(4+10)$$
$$= 85.14\% \text{ non-defective product.}$$

The utility scores are as follows:

$U_{Quality}$ = 0.50 and U_{Cost} = 0.75

For quality the score is "average" and "above average" for cost.

This solution is shown to the decision maker and the decision maker determines if this is the best comprise solution. If not, the decision maker provides new utility information. Assume the decision maker wants to improve quality. To improve quality the decision maker must compromise cost. The new utility information is provided in Exhibit 6.

Exhibit 6. Utility Functions for Second Iteration.

Performance Level	Quality Level	Total Cost	Utility Value
Poor	38.0%	$2,500.00	0
Below Average	63.0%	$1,500.00	0.25
Average	70.0%	$1,000.00	0.50
Above Average	88.0%	$500.00	0.75
Exceptional	90.0%	$240.00	1.00

The new solution is

x_1 = 50 units,
x_2 = 0 units,
x_3 = 0 units, and
x_4 = 10 units.

The values for the objective functions are:

$$\text{Cost} = (30x_1 + 24x_4) = (30 \times 50) + (24 \times 10)$$
$$=\$1,740.00$$

$$\text{Quality} = (0.98x_1 + 0.80x_4)/(x_1 + x_4)$$
$$= (0.98\times4 + 0.80\times10)/(4+10)$$
$$= 95.00\% \text{ non-defective product.}$$

The utility scores are as follows:

$U_{Quality}$ = 1.00 and U_{Cost} = 0.00

For quality the score is "exceptional" and "poor" for cost.

The process of showing a solution to the decision maker and then eliciting new information from the decision maker would continued until the decision maker finds the best compromise solution. The information provided by the decision makers in the form of utility functions does not require explicit utility information. The information is approximated based on the information provided.

Conclusions

The Multiple Objective Compromise Procedure has been previously used in an application to evaluate a hydrostatic bearing design problem. The method proved effective, but concerns lingered about possible bias on the part of the decision maker with respect to the initial single objective problems and the ranking of objectives in the iterative portion of the procedure. It is believed that estimating utility functions that represent a decision maker's attitude for each objective function will yield the following advantages:

- A smaller solution set that consists of a more realistic range of values, focusing on more probably values;
- The utility functions may be more representative of the decision maker's preferences than the desirability functions; and
- The utility functions include risk attitudes not incorporated in the desirability functions.

Future Work

The authors are currently developing a detailed methodology that incorporates estimated utility functions into the Multiple Objective Compromise Programming procedure to address the areas of possible bias that have been identified. Once the methodology has been finalized, the authors plan to test the method using graduate-level engineering and engineering management students to solve an example problem. Any necessary refinements to the methodology will be addressed and the decision making method will be applied to an industrial application.

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About the Authors

Sampson E. Gholston is an assistant professor at The University of Alabama in Huntsville. He has a Ph. D. from The University of Alabama in Huntsville in Industrial and Systems Engineering and a Masters from The University of Alabama in Industrial Engineering. His research interests include manufacturing and quality engineering, quality management, and applied statistics. He also worked for Saturn Corporation as a supplier quality engineer.

Alisha Youngblood is an assistant professor at The University of Alabama in Huntsville. She has a Ph. D. and Masters from The University of Arkansas in Industrial Engineering. Her research interests include engineering management, performance measurement, economic decision analysis, and logistics. She is an active member of IIE, ASEE, ASEM, and INFORMS.